Statistical Model for the Calculation of Conductance Variations of Memristive Devices

Nadine Dersch[1,2], Eduardo Perez[3,4], Christian Wenger[3,4], Mario Lanza[5], Kaichen Zhu[6], Mike Schwarz[1], Benjamín Iñíguez[2] and Alexander Kloes[1]
[1]THM Giessen, [2]URV Tarragona, [3]IHP Frankfurt(Oder), [4]BTU Cottbus, [5]NUS Singapore, [6]FDU Shanghai

***Abstract*—This paper presents a statistical model which calculates the expected conductance variations from device to device or from cycle to cycle of memristive devices. The mean read-out current and its standard deviation can be calculated for binary and multi-level devices. These values are important for simulating hardware-based artificial neural networks at circuit level and testing their functionality. Research into hardware-based artificial neural networks is important because they are energy-efficient. Furthermore to calculating the variations, the statistical model can be used to determine what influence the cumulative distribution function of switching has on the variations and which behavior provides the best results for the hardware-based artificial neural network. Some memristive devices exhibit multi-level behavior due to defects in the switching layer. The number of these defects and the optimal amount can be estimated.**

***Index Terms*—Memristive devices, statistical variations, binary, multi-level, artificial neural networks, cumulative distribution**

I. Introduction

The structure of artificial neural networks (ANNs) consisting of neurons and synapses originates from the human brain [1]. An energy-efficient implementation is possible with hardware-based ANN (Neuromorphic computing) [2] [3]. This requires devices that can imitate the function of human synapses. Promising candidates for this are memristive devices (MDs) such as resistive random access memory (RRAM) [4]. The MDs imitate the potentiation of synapse with their SET process and the depression of synapses with their RESET process [5]. After the SET process, the MDs are switched on and they are in the so-called low-resitive state (LRS). After the RESET process, they are in the so-called high-resistive state (HRS) and they are switched off [5]. There are two types of switching in MDs: binary or multi-level. The MDs exhibit variations from device-to-device (D2D) and from cycle-to-cycle (C2C) [6]. The human brain also exhibits the property of variations [7]. Before the hardware-based ANN is completely fabricated and measured, it is better to test their function using simulations. In the following, a statistical model (SM) is presented that makes it possible to determine the expected variations and the mean value of the current through MDs depending on a pulse-based programming. The variations can then be simulated at circuit level using the noise-based variability approach (NOVA) which is up to 1,000 times faster than the classical Monte Carlo simulation [8]. With NOVA, a fast simulation of an ANN with classification results is possible. [8]

II. The Statistical Model

The SM calculates the expected mean values of the read-out currents through MDs (corresponding to its conductance) and their variations. The SM assumes programming of the MDs via pulses as in [8]. The pulses can be varied in amplitude, pulse width of the programming voltage ($V_P PW$) and number (see **figure 1**). Here, the pulse width of the read-out voltage ($V_R PW$) is constant for every measurement. The cumulative distribution function (CDF) of switching of the MDs is needed for the SM. For example, such CDF can be calculated using [3]. Here, the CDF is determined via the measurements and a limit between HRS and LRS. The limit defines the transition from HRS to LRS or in reverse.

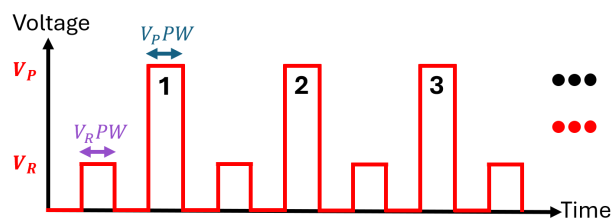

Fig. 1. Representation of the programming pulse (red), which consists of a programming voltage (V_P) and a read-out voltage (V_R). These values can be modified, as well as the number of pulses (black) and $V_P PW$ (blue). Here, $V_R PW$ is constant (purple).

The situation is: A large number N of MDs is programmed via the SET process with m pulses (starting point HRS and switching to LRS). After a number of m_0 pulses, some of the N programmed MDs are still in the HRS (N_{HRS}) and the others have switched to the LRS (N_{LRS}). This information is described by the CDF. The distribution followed by both LRS and HRS ranges is Gaussian [8]. This results in a mean value and a standard deviation (STD) of the currents in LRS and HRS. Δ_{LRS} and Δ_{HRS} are random values indicating the distance to the mean value. When using the RESET process, the starting point and final point are inverted. **Figure 2** shows this description schematically for the SET process.

The mean value of the current $\bar{I}(m_0)$ after m_0 pulses is calculated by using the parameters from **figure 2** via

$$\bar{I}(m_0) = \sum_{1}^{N_{HRS}} \frac{\bar{I}_{HRS} + \Delta_{HRS}}{N} + \sum_{1}^{N_{LRS}} \frac{\bar{I}_{LRS} + \Delta_{LRS}}{N}, \quad (1)$$

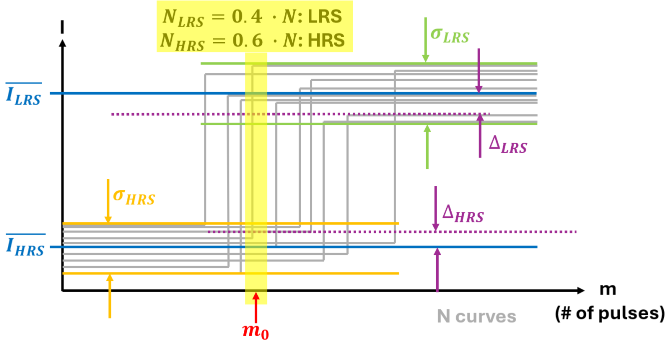

Fig. 2. Schematic representation of the theory of the SM: N measurement curves over m pulses that are either in the HRS or LRS are shown in gray (SET). The STD (orange and green) and the mean values (blue) are shown for HRS and LRS. Δ_{HRS} and Δ_{LRS} (purple) are the deviations of a random curve from the mean value. Here, 60% of the curves are in HRS and 40% in LRS at the pulse number m_0 (red).

and as the mean values of HRS and LRS are required here, Δ_{LRS} and Δ_{HRS} can be set to zero. The probability variable $F(m_0)$ after m_0 pulses from the CDF is then required. This changes **equation (1)** to

$$\bar{I}(m_0) = (1 - F(m_0)) \cdot \bar{I}_{HRS} + (F(m_0)) \cdot \bar{I}_{LRS}, \quad (2)$$

which can be used to determine the expected mean value. The variance of the current $\bar{I}(m_0))$ after m_0 pulses is calculated using

$$\sigma_I^2(m_0) = \frac{1}{N} \cdot \sum_{1}^{N} \left(I(m_0) - \bar{I}(m_0)\right)^2. \quad (3)$$

Equation (3) can then be divided into a HRS and a LRS part and transformed using the second binomial formula:

$$\begin{aligned} \sigma_I^2(m_0) = \frac{1}{N} \cdot \sum_{1}^{N_{HRS}} 2\Delta_{HRS}(m_0) \cdot (\bar{I}_{HRS} - \bar{I}(m_0)) + \\ \frac{1}{N} \cdot \sum_{1}^{N_{HRS}} \left(\bar{I}_{HRS}(m_0) - \bar{I}(m_0)\right)^2 + \frac{1}{N} \cdot \sum_{1}^{N_{HRS}} \Delta_{HRS}^2(m_0) + \\ \frac{1}{N} \cdot \sum_{1}^{N_{LRS}} \left(\bar{I}_{LRS}(m_0) - \bar{I}(m_0)\right)^2 + \frac{1}{N} \cdot \sum_{1}^{N_{LRS}} \Delta_{LRS}^2(m_0) + \\ \frac{1}{N} \cdot \sum_{1}^{N_{LRS}} 2\Delta_{LRS}(m_0) \cdot (\bar{I}_{LRS} - \bar{I}(m_0)) \end{aligned} \quad (4)$$

Using the probability variable $F(m_0)$, considering that Δ_{LRS} and Δ_{HRS} are in connection with $\bar{I}_{LRS}$ and $\bar{I}_{HRS}$ equal to zero, the **equation (4)** is simplified to

$$\begin{aligned} \sigma_I^2(m_0) = (1 - F(m_0)) \cdot \left(\left(\bar{I}_{HRS} - \bar{I}(m_0)\right)^2 + \sigma_{HRS}^2 \right) + \\ F(m_0) \cdot \left(\left(\bar{I}_{LRS} - \bar{I}(m_0)\right)^2 + \sigma_{LRS}^2 \right), \end{aligned} \quad (5)$$

which can be used to calculate the expected variation. The final equations used in the model are **equation (2)** and **equation (5)**.

III. Verification with Binary Memristive Devices

The HfO_2-based MDs from [2] are used here to verify the SM. This is a binary MD ("0" or "1"). Inside the MD is a conductive filament (CF). The MD is in the LRS when the CF is formed and it is in HRS when the CF is broken. The read-out voltage is 0.2 V, the programming voltage is varied from 0.6 V to 1.2 V, the $V_P PW$ 100 ns, 1 µs and 10 µs are set, and a number of 100 pulses is applied to the MD. A total of 128 different MDs are tested. This determines the D2D variability and is done for SET and RESET for all combinations.

For comparison, the mean value and the STD of the current of the measurement is determined over 128 curves [2]. **Figure 3 (a)** and **(c)** shows example measurements.

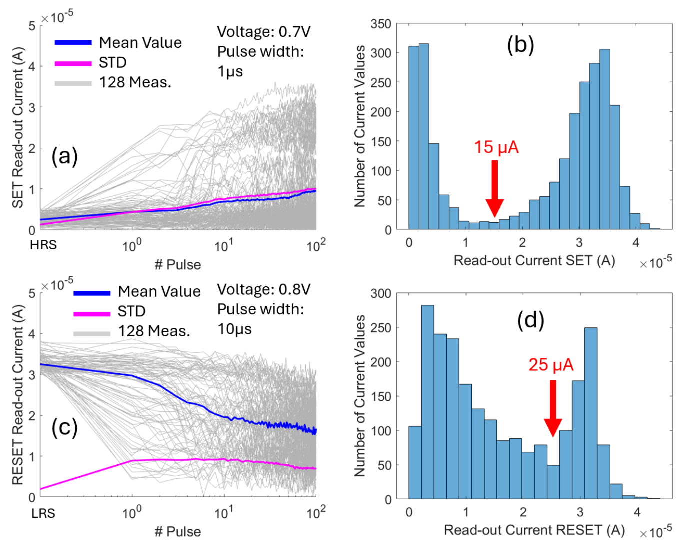

Fig. 3. D2D measurements of 128 MDs (gray) and their mean value (blue) and STD (pink) with a 0.7 V and 1 µs pulse (SET) in (a) and with a −0.8 V and 10 µs pulse (RESET) in (c). Current distribution over all measurement data after 100 pulses from SET in (b) and RESET in (d), whose minimum is marked in red (limit).

The CDF is then determined from the measurements, using a limit between HRS and LRS as a fitting parameter. For this purpose, the distribution of all currents after 100 pulses was determined as shown in **figure 3 (b)** and **(d)**. The minimum between HRS and LRS was set as the value for the limit. For SET is the limit 15 µA and for RESET is the limit 25 µA.

The values $\bar{I}_{LRS}$, $\bar{I}_{HRS}$, σ_{LRS} and σ_{HRS} can be determined from the initial state and the result after the last pulse. **Figure 4** shows the results by considering different programming voltages (±0.6 V to ±1.2 V). For reasons of clarity, all seven mean values are shown, but only four STDs (cases where direct switching after the first pulse and no switching after 100 pulses are omitted). For the testing of the scalability of the pulse width, the $V_P PW$s 100 ns, 1 µs and 10 µs are set. The results are shown in **figure 5**.

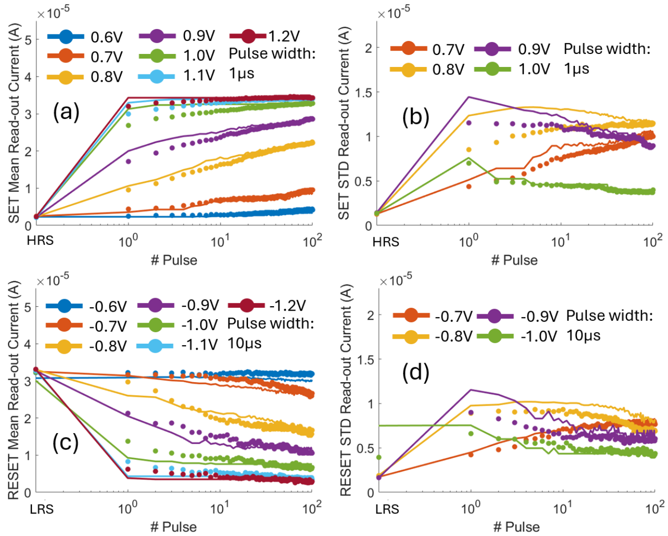

Fig. 4. (a) - (d) Model (lines) vs. measurements (points). Mean values of the voltages $\pm 0.6\,\mathrm{V}$ to $\pm 1.2\,\mathrm{V}$ and $V_P PW$ $1\,\mu\mathrm{s}$ (SET) in (a) and $V_P PW$ $10\,\mu\mathrm{s}$ (RESET) in (c). STD of the voltages $\pm 0.7\,\mathrm{V}$ to $\pm 1.0\,\mathrm{V}$ and $1\,\mu\mathrm{s}$ (SET) in (b) and $10\,\mu\mathrm{s}$ (RESET) in (d).

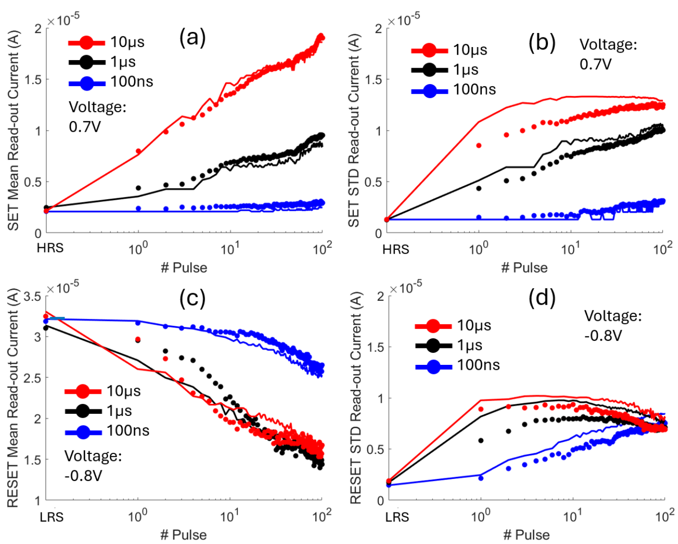

Fig. 5. (a) - (d) Model (lines) vs. measurements (points). Here the $V_P PW$ is $100\,\mathrm{ns}$, $1\,\mu\mathrm{s}$ and $10\,\mu\mathrm{s}$. The voltage for SET is $0.7\,\mathrm{V}$ and for RESET is $-0.8\,\mathrm{V}$. The mean values are shown in (a) for SET and in (c) for RESET. The STDs are shown in (b) for SET and in (d) for RESET.

Figure 4 and **5** show good agreement between the SM and the measurements. Further, this provides scalability over the whole voltage and pulse width ranges. The results show that the equations perform statistical calculations correctly.

IV. Application of Statistical Model in Theory

One application of the SM is to determine the influence of the CDF or to find out which switching behavior is beneficial for the performance of the ANN. A fast way to analyze a hardware-based ANN at circuit level is via NOVA [8]. NOVA simulates the variations of the MDs and needed their mean value and STD. The curve of the CDF has an influence on the resulting mean values and STD. These have a direct influence on the implementation of the ANN and the success of a good classification. From this, conclusions can be made regarding which CDF is preferable for the ANN in order to achieve the best possible results. Tests were thus carried out with a variation of the CDF. Results with HfO_2-based MDs are shown in **figure 6**. For the theory, a classical exponential distribution was used and fitted to the measurements. It should be noted that the CDF of the measurement achieves a maximum at 75%. This had to be taken into account. 25% of the MDs will not switch even after another 100 pulses and are effectively rejections. CDF was then adjusted in theory so that the rejections decrease to 10% and increase to 50%. A more or less steeper slope of the CDF was also investigated. The results show that the number of rejections has a direct influence on the maximum mean value and value of the STD. Whereas the slope only determines the pulse number at which the maximum mean value is reached and where the STD reaches the maximum value.

V. Application of the Statistical Model for Multi-Level Memristive Devices

A TiO_2-based MD is used here. The dimensions of the device structure are $40\,\mathrm{nm}$ Au / $10\,\mathrm{nm}$ Ti / $10\,\mathrm{nm}$ TiO_2 / $40\,\mathrm{nm}$ Au with a device size of $5\,\mu\mathrm{m} \times 5\,\mu\mathrm{m}$ cross-point structure. Like the binary MDs, the multi-level MDs have a CF. However, there are multiple defects in the oxide, which allow for the formation of multiple weak CFs. Depending on the total number N_{Defect} and its fraction at which a CF has been formed, the current flowing increases or decreases.

Three different programming pulses, each with a $V_P PW$ of $150\,\mathrm{ms}$, are tested for the MD. The programming voltage for RESET is $-3.2\,\mathrm{V}$ and for SET is $3.6\,\mathrm{V}$ or $3.8\,\mathrm{V}$. In addition, the read-out voltage is $0.2\,\mathrm{V}$ and the number of pulses is 40 or 100. The measurement is repeated 10 to 20 times on the same MD. This determines the C2C variability. **Figure 7 (a)** shows these measurement results. To calculate the expected mean value and variance, the **equation (2)** and **equation (5)** are used for a single defect. To determine the total device current comprised of adding the currents of N_{Defect} binary switching devices in parallel, the mean value is obtained from $N_{Defect} \cdot \bar{I}(m_0)$ and the standard deviation from $\sqrt{N_{Defect} \cdot \sigma_I^2(m_0)}$. The comparison between the SM and the measured data can be seen in **figure 7 (b)** and **(c)**.

The comparison of the SM and the measurement data shows that the results are very similar and the calculation is reasonable. For the setting $3.6\,\mathrm{V}/-3.2\,\mathrm{V}$ (40 pulses), N_{Defect} is 150/100 (which is equivalent to a defect density of $6 \times 10^8\,\mathrm{cm}^{-2}$ / $4 \times 10^8\,\mathrm{cm}^{-2}$). This ensures that the maximum current decreases with the new SET process. The current value for the first measurement is greater than for the last measurement. The same effect can be observed with setting $3.6\,\mathrm{V}/-3.2\,\mathrm{V}$ (100 pulses), when N_{Defect} is 235/130 ($9.4 \times 10^8\,\mathrm{cm}^{-2}$ / $5.2 \times 10^8\,\mathrm{cm}^{-2}$). N_{Defect} is 100/100 by

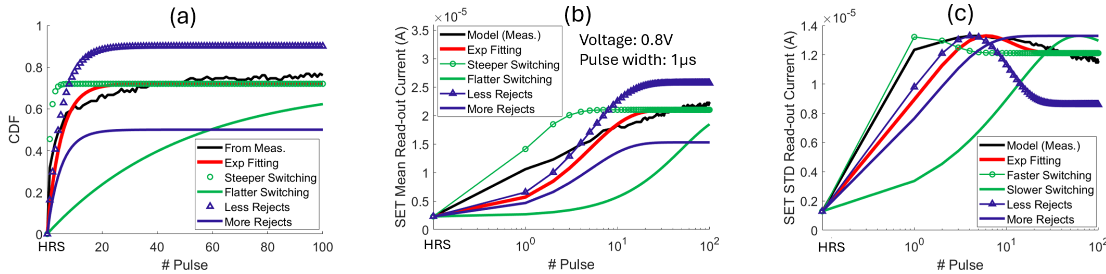

Fig. 6. The CDF is determined from the measurement data (black), fitted via an exponential distribution (red), receives a steeper switching behavior (green, dots), receives a flatter switching behavior (green, line), receives less rejects (blue, triangle) and receives more rejects (SET with 0.8 V and 1 µs). (a) CDF. (b) Mean Value. (c) STD.

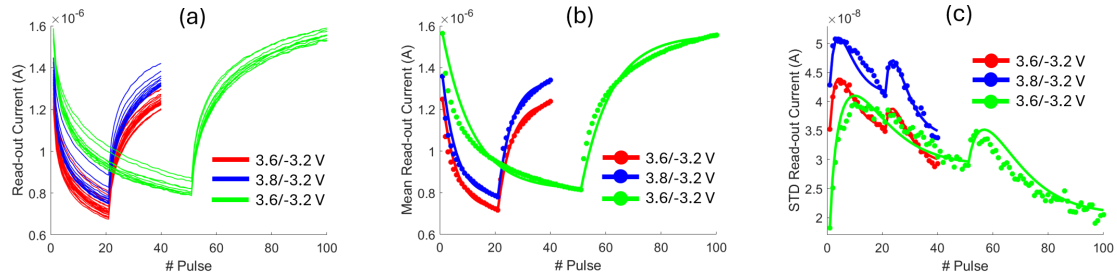

Fig. 7. (a) C2C measurements. (b) Mean value and (c) STD of the model (line) and of the measurements (points).

setting $3.8\,\text{V}/-3.2\,\text{V}$ (40 pulses). Here, the same number of defects shown switching for the RESET and SET process, which could be forced by the increased voltage, which has a stronger effect. The SM can therefore be used to determine how many defects are present in the MD and how the number changes over several programming processes (change of the rejections). The CDF could also be considered in more detail.

VI. Conclusion

The SM can be used to calculate the expected mean value and STD of MDs. It is applicable for C2C and D2D variability. The developed equations for the mean value of the current at pulse number m_0 ($\bar{I}(m_0)$) and the standard deviation of the current at pulse number m_0 ($\sqrt{\sigma_I^2(m_0)}$) are applicable for binary and multi-level MDs. However, attention must be paid to the physical effects here. In the case of the binary HfO_2-based MDs, the total current could be considered directly with the equations. With the analog TiO_2-based MDs, on the other hand, the equations were used to determine the current from one defect and the number of all defects still had to be taken into account for the total current. The simulation results of two technologies were investigated and agreed very well with the measured data (for the mean value and the STD). The SM also shows good scalability over the programming voltage and the pulse width range. Therefore, the SM allows to quickly and efficiently calculate the mean value and STD of the read-out current of the MDs, allowing by using NOVA a simulation of energy-efficient hardware-based ANNs at circuit level. Due to the speed advantage, parasitic elements can be considered in the circuit, and their effect together with the influence of the CDF of switching on the ANN performance can be estimated. The SM can be used to determine which settings lead to the best results and how the real behavior of the MDs must be adjusted accordingly.

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